Rank Tests for Corrupted Linear Models[*]

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Abstract

For some variants of regression models, including partial, measurement error or error-in-variables, latent effects, semi-parametric and otherwise corrupted linear models, the classical parametric tests generally do not perform well. Various modifications and generalizations considered extensively in the literature rests on stringent regularity assumptions which are not likely to be tenable in many applications. However, in such non-standard cases, rank based tests can be adapted better, and further, incorporation of rank analysis of covariance tools enhance their power-efficiency. Numerical studies and a real data illustration show the superiority of rank based inference in such corrupted linear models.

Key words: Latent variable; Measurement error; Mixed regression model; Partially linear model; Rank analysis of covariance; Rank analysis of variance; Rank test of linear hypothesis

1 Introduction

Classical linear regression models induce some stringent additivity, linearity, homoscedasticity and normality assumptions which may not be tenable in many applications giving rise to the so called corrupted linear models where one or more of these assumptions may not be tenable. In simple nonparametric linear models, the normality assumption has been dispensed with in favor of a more general class of continuous distributions. Yet, in more contemporary applications in biomedical, clinical and genomics studies,

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the very assumption of linearity may be questionable. Sans such a linear setup, the performance of rank based testing procedures may be generally far better than their strict parametric counterparts. Our contemplated corrupted linear models relate to this scenario where the basic linearity assumption is vitiated by possible error-in-variables, measurement errors, possible latent effects, and the so called random effects and mixed effects; even partial linear models and some semi-parametric models belong to this contemplated class. For example, Fuller (1987) has detailed a large class of models which can be classified as measurement error or error-in-variable models; some genuine identifiability issues may crop-up in the use of standard parametric inference. Another variation is the usual regression models with stochastic predictors whose possible non-normal distribution can create stumbling blocks to the adaption of standard parametric methods. In addition such stochastic predictors may not be linearly related with the primary response variable. The impact of such nonregular setups on statistical tests has been considered by Ghosh and Sen (1971), followed by more general treatise by others. In a semi-parametric setup, partial linear models were introduced mostly during the 1980s and 1990s (Heckman (1986), Speckman (1988), Khuri, Mathew and Sinha (1988), Chen (1988), Gao (1995), He and Shi (1996), Liang et al. (1999), Hardle et al. (2000), He and Liang (2000), and Boente and Rodriguez 2006, among others). Incorporation of measurement errors in this setup evolved first in nonlinear models (Carroll et al. (2006)) and then in nonparametric setups only in the past decade. For nonlinear models one may try to mimic the linear model setups with linear or quadratic approximations, but again those may call for a second source of non-robustness arising from such possibly inadequate approximations. Motivated by this diversity of models and the need for a unified view of such nonstandard or corrupted linear models, the present study mainly aims to introduce such corrupted linear models in a more general setup, exhibit the supremacy of rank based tests and illustrate its adaptability in some real applications.

Consider a *semiparametric partially linear* model where a real response Y is regressed to a set of observable covariates $\mathbf{x}$ and further depends on some possibly unobservable $\mathbf{Z}$ in the form:

$$Y_i = \beta_0 + \mathbf{x}_i^\top \boldsymbol{\beta} + \nu(\mathbf{Z}_i) + e_i, \quad i = 1, \ldots, n, \tag{1.1}$$

where the $\mathbf{x}_i$ are known (non-stochastic) p-vectors, not all the same, $\mathbf{Z}_i$ is a stochastic q-vector covariate ($q \geq 1$), and the form of the function $\nu(\mathbf{Z}_i)$ is unspecified. Moreover, the $\mathbf{Z}_i$ may be observable, partially observable or unobservable; in the latter case, they lead to latent effects models. If the $\mathbf{Z}_i$ are observable, eventually with measurement errors, (1.1) relates to a partially linear and measurement error model. he unknown $\nu(.)$ links (1.1) to the semiparametric model. A big advantage of the rank procedure is that it avoids a nonparametric estimation of unknown $\nu(.)$. The literature recommends the functional estimation procedures, using various smoothing tools; but they demand smoothness assumptions, while they usually result in slower rates of convergence than the rank procedures. We refer to Heckman (1986), Speckman (1988), Chen (1988), He and Shi (1996), He and Liang (2000), Bianco et al. (2006), Boente and Rodriguez (2006), among other works. The Härdle et al. (2000) monograph is noteworthy in this context.

An alternative approach is a transformation of variables in regression problems

which achieves linearity or normality; but this usually sacrifices the homoscedasticity condition. The heteroscedastic models and models with measurement errors were intensively treated in the literature; we refer to Fuller (1987), Cheng and van Ness (1999) and Carroll et al. (2006), and to additional references cited therein.

In contrast to the above methods, we put the main emphasis on nonparametric tests based on rank statistics. They are valid also for non-normal error distributions, do not demand the finite variances, and their asymptotic forms typically have the standard rate of convergence $n^{-1/2}$. The smoothing techniques as B-splines and kernel smoothing, which are commonly used for estimation in the semiparametric linear models, generally require a large n and result in a slower rate of convergence than $n^{-1/2}$.

The problem of testing the monotonicity of regression was considered by [12], who used a nonparametric approach in a semiparametric setup. These models can be sometimes reduced to (1.1) by suitable reformulation.

We often want to test the null hypothesis of no or partial regression of Y on $\mathbf{x}$, treating β_0 and $\nu(\cdot)$ as nuisance parameters and functions, respectively. The statistical interest is then confined to the fixed-effect parameter $\boldsymbol{\beta}$, regarding $\nu(\cdot)$ as a nuisance function, similarly as in the [8] proportional hazard model. More precisely, we want to test

$$\mathbf{H}_0 : \ \boldsymbol{\beta} = \mathbf{0} \quad vs \quad \mathbf{H}_1 : \ \boldsymbol{\beta} \neq \mathbf{0} \tag{1.2}$$

with nuisance β_0 and $\nu(\cdot)$.

Although $\nu(\cdot)$ is unspecified in (1.1), it is of interest to distinguish two cases according as the covariate $\mathbf{Z}$ is observable or not. If $\mathbf{Z}$ is unobservable, (1.1) corresponds to the *latent effects* model, although in the usual linear model setup, $\nu(\mathbf{Z})$ is taken to be a linear functional, whereas in (1.1) it is unspecified. If $\mathbf{Z}_i$'s are observable and regarded as identically distributed random variables with some unspecified distribution and independent of the error e_i, then letting $e_i^* = e_i + \nu(\mathbf{Z}_i)$ we may still claim that the e_i^* are i.i.d. random variables. However, their distribution function is unlikely to be normal even if the $\mathbf{Z}_i$ were normally distributed; this is specially because of the unspecified nature of $\nu(\cdot)$. On the other hand, since the e_i^* are independent identically distributed random variables, the classical nonparametric rank based tests are adaptable. This naturally suggests that nonparametric tests based on rank statistics would have better scope as well as power properties.

There is a much better perspective if the $\mathbf{Z}_i$, though stochastic, are observable. Unlike the parametric analysis of covariance (ANOCOVA) the assumption of linearity of regression is not necessary in the nonparametric ANOCOVA approach. Quade (1969) considered a rank ANOCOVA procedure based on the rank sum statistics, and that was extended immediately to general scores tests in more general models by Puri and Sen (1971) where earlier references are also cited. Even the work of Ghosh and Sen (1971) is closely related to this aspect of rank tests. In this context, by virtue of the fact that ranks are invariant under any strictly monotone transformation on the covariates, the linearity of the regression on covariate may no longer be necessary, and the resulting rank ANOCOVA tests are therefore much more robust than their parametric counterparts and typically have greater power than nonparametric rank ANOVA tests which ignore the covariates. This improvement comes out of the fact that the joint distribution of the coordinatewise rank statistics is typically close to a multinormal one

and that validates the use of ANOCOVA tools even when the underlying form of $\nu(.)$ is nonlinear. Even more, the rank tests are still applicable if the $\mathbf{Z}_i$ are observable, but subject to measurement errors as in the model considered by Nummi and Möttönen (2004); then the e_i^* are still i.i.d. random variables, though with some other distribution function. This shows an advantage of the nonparametric analysis of covariance procedures comparing with other methods.

In a general regression setup where the regressors are stochastic, Ghosh and Sen (1971) modified the usual rank tests for testing the hypothesis of no regression, and in the measurement error model, Jurečková et al. (2010) considered suitable rank tests. In both the cases, the hypothesis of no regression generates the same invariance structure which validates the conventional rank tests. This does not, however, exploit the stochastic nature of the regressors to the fullest extent. In the present study, it is demonstrated that the incorporation of rank analysis of covariance tools in this more complex setup (1.1) yields rank tests which have better performance characteristics. To emphasize this enhanced efficiency, extensive numerical studies on simulated as as well as a real data set are carried out. Section 2 is devoted to the preliminary notions and description of the methods. Section 3 deals with the partially linear model with i.i.d. nuisance covariates. Section 4 is devoted to rank analysis of covariance in partially linear models, and Sections 5 and 6 provide numerical illustrations, both on simulated and real data.

2   Preliminary notion

We motivate our statistical models through an interesting case studied by Nummi and Möttönen (2004). They described a computer-based forest harvesting technique in Scandinavia, where the tree stems are converted into smaller logs and the stem height and diameter measurements are taken at fixed intervals. The harvester receives the length and diameter data at the ith stem point from a sensor, and a measuring and computing equipment enables a computer-based optimization of crosscutting. Nummi and Möttönen (2004) consider the model of regression dependence of the stem diameter measurement y_i on the stem height measurement x_i at the ith stem point, $i = 1, \ldots, n$. The problem of interest is the prediction for y_i and the testing of hypotheses on the parameters of the model; but both the stem diameter and the stem height contain measurement errors. On top of that the volume of the stem may not be linearly related to its diameter, rather it is more likely to be related to its height and the cross-section which may be roughly proportional to the square of the diameter.

There are many other similar problems which can be described by partially linear regression models of the type (1.1) where $\mathbf{x}_i$ is a p-vector covariate, $\mathbf{Z}_i$ is a q-vector covariate, the function $\nu(\cdot)$ is unknown, and the model error e_i is independent of $(\mathbf{x}_i, \mathbf{Z}_i)$, $i = 1, \ldots, n$. It means that the response variable Y_i depends on variable $\mathbf{x}_i$ in a linear way but is still related to another independent variables $\mathbf{Z}_i$ in an unspecified form, $i = 1, \ldots, n$. This model, along with the measurement errors model, are flexible and enable to model various situations with latent variables present.

In (1.1) we assume that the independent errors $e_1, \ldots, e_n$ are identically distributed

according to an unknown distribution function F, and that $\boldsymbol{\beta}^\top = (\beta_1, \ldots, \beta_p)$, $\boldsymbol{\beta}^* = (\beta_0, \boldsymbol{\beta}^\top)^\top$ are unknown parameters. The function $\nu(\cdot)$ is unknown and $\mathbf{Z}_i$ are additional covariates; if they are unobservable, then all $\nu(\mathbf{Z}_i)$, $i = 1, \ldots, n$ are latent random variables. The rank tests of for this situation with unobservable $\mathbf{Z}_i$ are studied in Section 3. If $\mathbf{Z}_i$'s are observable, we can use this additional information even if $\nu(\cdot)$ remains unknown, and apply the methods of the rank analysis of covariance; very important is that this method is successful even when $\mathbf{Z}_i$ itself is affected by a measurement error (Section 4).

Our interest is to find how the rank tests of hypothesis $\mathbf{H}_0$ in (1.2) behave in the described situations and to demonstrate their superiority to other methods. They are distribution free and avoid an estimation of nuisance $\nu(\cdot)$, which would always worsen the rate of convergence of the whole procedure. The numerical study in Section 5 illustrates the good behavior of the rank tests in situations with various uncertaintes.

3   Partially linear model with i.i.d. latent variables

Consider the partially linear model (1.1) and the problem of testing the hypothesis $\mathbf{H}_0 : \boldsymbol{\beta} = \mathbf{0}$, with β_0 and function $\nu(\cdot)$ unknown, the $\mathbf{Z}_i$ (scalar or vector random variables) unobservable. The model can be rewritten as

$$Y_i = \beta_0 + \mathbf{x}_i^\top \boldsymbol{\beta} + e_i^*, \ e_i^* = e_i + \nu(\mathbf{Z}_i), \ i = 1, \ldots, n. \tag{3.1}$$

The regression matrix $\mathbf{X} = \mathbf{X}_n$ in model (1.1) is of order $n \times p$ with the rows $\mathbf{x}_i$, $i = 1, \ldots, n$. Denote $\mathbf{X}_n^0$ the matrix with the rows $\mathbf{x}_i - \bar{\mathbf{x}}_n$, $i = 1, \ldots, n$, and assume that it satisfies

$$\mathbf{Q}_n = \frac{1}{n} \mathbf{X}_n^{0\top} \mathbf{X}_n^0 = \frac{1}{n} \sum_{i=1}^n (\mathbf{x}_i - \bar{\mathbf{x}}_n)(\mathbf{x}_i - \bar{\mathbf{x}}_n)^\top \to \mathbf{Q} \ \text{ as } \ n \to \infty, \tag{3.2}$$

$$n^{-1} \max_{1 \le i \le n} \left\{ (\mathbf{x}_i - \bar{\mathbf{x}}_n)^\top \mathbf{Q}_n^{-1} (\mathbf{x}_i - \bar{\mathbf{x}}_n) \right\} \to 0 \ \text{ as } \ n \to \infty$$

where $\mathbf{Q}$ is a positive definite $p \times p$ matrix.

Assume that the distribution function F of the errors e_i has an absolutely continuous density f and finite Fisher information $\mathcal{I}(f) = \int_{\mathbb{R}} \left(\frac{f'(z)}{f(z)} \right)^2 dF(z) < \infty$. Assume that $\mathbf{Z}_1, \ldots, \mathbf{Z}_n$ are i.i.d.; let G be the joint distribution function of $\nu(\mathbf{Z}_i)$, $i = 1, \ldots, n$. It is unknown, we only assume that it has an absolutely continuous density g and finite Fisher information $\mathcal{I}(g)$. Moreover, let H denote the distribution function of e_i^*, $i = 1, \ldots, n$. Because e_i^* is more dispersed than e_i, then $\mathcal{I}(h) \le \mathcal{I}(f)$, where $\mathcal{I}(h)$ is the Fisher information of H, with the equality if $\nu(\mathbf{Z}_i) = 0$ with probability 1 (see Hájek et al. (1999)).

Let $R_1, \ldots, R_n$ be the ranks of $Y_1, \ldots, Y_n$. The rank tests of $\mathbf{H}_0 : \boldsymbol{\beta} = \mathbf{0}$, both in models (1.1) and (3.1), are based on the vector of linear rank statistics $\mathbf{S}_n \in \mathbb{R}^p$,

$$\mathbf{S}_n = n^{-1/2} \sum_{i=1}^n (\mathbf{x}_i - \bar{\mathbf{x}}_n) a_n(R_i) \tag{3.3}$$

where the scores $a_n(i)$ are generated by nondecreasing, square integrable score function $\varphi : (0, 1) \mapsto \mathbb{R}^1$ in either of the following two ways:

$$a_n(i) = \mathbb{E}\varphi(U_{n:i}), \tag{3.4}$$

$$a_n(i) = \varphi\left(\frac{i}{n+1}\right), \quad i = 1, \ldots, n,$$

and $U_{n:1} \leq \ldots \leq U_{n:n}$ are the order statistics corresponding to the sample of size n from the $R(0, 1)$ distribution. The test criterion for $\mathbf{H}_0$ is the quadratic form in $\mathbf{S}_n$,

$$\mathcal{T}_n^2 = (A(\varphi))^{-2}\, \mathbf{S}_n^\top \mathbf{Q}_n^{-1} \mathbf{S}_n \tag{3.5}$$

where

$$A^2(\varphi) = \int_0^1 (\varphi(t) - \bar\varphi)^2 dt, \quad \bar\varphi = \int_0^1 \varphi(t) dt. \tag{3.6}$$

and because the ranks are distribution free, its asymptotic null distribution is χ^2 with p degrees of freedom, and the nonlinear regressor does not cause any bias.

On the other hand, the asymptotic distributions of $\mathcal{T}_n^2$ under the local alternative

$$\mathbf{H}_n : \quad \boldsymbol{\beta} = \boldsymbol{\beta}_n = n^{-1/2}\boldsymbol{\beta}^*, \quad \mathbf{0} \neq \boldsymbol{\beta}^* \in \mathbb{R}^p \quad \text{fixed}, \tag{3.7}$$

are the noncentral χ^2 distributions with generally different noncentrality parameters. The relative asymptotic efficiency of the test in the presence of the nonlinear covariate with respect to that in a genuinely linear model is given in the following theorem:

Theorem 1 *Let $\mathcal{T}_n^2$ be the test criterion (3.5) for $\mathbf{H}_0$ and $\mathcal{T}_{n0}^2$ be its special case corresponding to $P(\nu(\mathbf{Z}) = 0) = 1$. Then*

(i) *Under $\mathbf{H}_0$, both $\mathcal{T}_n^2$ and $\mathcal{T}_{n0}^2$ have asymptotically χ^2 distribution with p degrees of freedom, as $n \to \infty$.*

(ii) *The asymptotic relative efficiency of $\mathcal{T}_n^2$ with respect to $\mathcal{T}_{n0}^2$ under the local alternative (3.7) is*

$$e(\mathcal{T}_n^2, \mathcal{T}_{n0}^2) = \left(\frac{\gamma(\varphi, h)}{\gamma(\varphi, f)}\right)^2 = \left(\frac{\int_0^1 h(H^{-1}(t)) d\varphi(t)}{\int_0^1 f(F^{-1}(t)) d\varphi(t)}\right)^2 \tag{3.8}$$

where f, F are the density and distribution function of e_1 in model (1.1), h, H are the same for e_1^* in model (3.1), and where

$$\gamma(\varphi, h) = \int_0^1 \varphi(t)\varphi(t, h) dt, \quad \varphi(t, h) = -\frac{h'(H^{-1}(t))}{h(H^{-1}(t))}, \tag{3.9}$$

and similarly for $\gamma(\varphi, f)$.

Proof. By Hájek et al. (1999), Sections V.1.5 and V.1.6, we have under $\mathbf{H}_0$ as well as under $\mathbf{H}_n$

$$\|\mathbf{Q}_n^{-1/2}[\mathbf{S}_n - \widetilde{\mathbf{L}}_n]\| = o_p(1) \quad \text{as} \quad n \to \infty \tag{3.10}$$

where

$$\widetilde{\mathbf{L}}_n = n^{-1/2} \sum_{i=1}^{n} (\mathbf{x}_i - \bar{\mathbf{x}}_n)\varphi(H(Y_i))$$

here $U_{n1}, \ldots, U_{nn}$ are the random samples from the uniform $(0,1)$ distribution. Hence, both $\mathcal{T}_n^2$ and $\mathcal{T}_{n0}^2$ are asymptotically χ^2 distributed with p degrees of freedom under $\mathbf{H}_0$. Under $\mathbf{H}_n$, the asymptotic distribution of $\mathcal{T}_n^2$ is the noncentral χ^2 with p degrees of freedom and with the noncentrality parameter

$$\Delta_H = \boldsymbol{\beta}^{*\top}\mathbf{Q}\boldsymbol{\beta}^* \, \frac{\gamma^2(\varphi, H)}{A^2(\varphi)}, \tag{3.11}$$

while $H \equiv F$ if $\nu(Z) = 0$ with probability 1. This yields (3.8) as the relative asymptotic efficiency (ARE) of the test $\mathcal{T}_n^2$ with respect to the test $\mathcal{T}_{n0}^2$. $\qquad\square$

For the special case of Wilcoxon scores, it follows that $\mathrm{e}(\mathcal{T}_n^2, \mathcal{T}_{n0}^2) \leq 1$, with the equality sign holding when $\nu(\mathbf{Z}) = 0$ with probability 1. Similar inequality holds for the median test, if f and g [density of $\nu(\mathbf{Z})$] are symmetric around 0 and f is unimodal, because then $\gamma(\varphi, h) = h(0) \leq f(0) = \gamma(\varphi, f)$, with the equality sign holding when $\nu(\mathbf{Z}) = 0$ with probability 1. For general scores, under star-shaped ordering of f and h (Doksum (1969), Bickel and Lehmann (1979)), it follows that $\mathrm{e}(\mathcal{T}_n^2, \mathcal{T}_{n0}^2) \leq 1$. If the test with score function φ is asymptotically optimal for f, i.e. if $\varphi(t) = \varphi(t, f)$, $0 < t < 1$, then $\mathrm{e}(\mathcal{T}_n^2, \mathcal{T}_{n0}^2) \leq \frac{\mathcal{I}(h)}{\mathcal{I}(f)} \leq 1$. In the general case,

$$\mathrm{e}(\mathcal{T}_n^2, \mathcal{T}_{n0}^2) \leq \frac{\mathcal{I}(h)A^2(\varphi)}{\gamma^2(\varphi, f)}.$$

It may be of interest whether there is a positive lower bound to (3.8). However, allowing the dispersion of $\nu(\mathbf{Z})$ to be large compared to that of e, it can be shown that under the same conditions as in above, (3.8) can be made arbitrarily close to 0. Thus, too much of latent effects can affect the efficacy of rank tests; it is similar in the parametric case if $\sigma_{\nu(\mathbf{Z})}^2/\sigma_e^2$ is large; then the latent-effects model lose the efficacy.

Besides the presence of a nonlinear nuisance regressor, the Y_i can be further affected by an additive measurement error. Hence, instead of Y_i we observe $\widetilde{W}_i = Y_i + V_i$, $i = 1, \ldots, n$, where the random errors $V_1, \ldots, V_n$ are assumed to be i.i.d. and independent of Y_i, $\mathbf{x}_i$, $\mathbf{Z}_i$, $i = 1, \ldots, n$. Their distribution (say $\tilde{G}$) is unknown, we only assume that it has an absolutely continuous density $\tilde{g}$. Then the model (1.1) can be further rewritten in the form

$$\widetilde{W}_i = \mathbf{x}_i^\top \boldsymbol{\beta} + \tilde{e}_i, \; \tilde{e}_i = e_i + \nu(\mathbf{Z}_i) + V_i, \; i = 1, \ldots, n.$$

Let $\tilde{R}_1, \ldots, \tilde{R}_n$ denote the ranks of $\widetilde{W}_1, \ldots, \widetilde{W}_n$. Under $\mathbf{H}_0$, they are independent and identically distributed, hence

$$P\Big((\tilde{R}_1, \ldots, R_n) = (r_1, \ldots, r_n)\Big) = \frac{1}{n!}$$

for every permutation $(r_1, \ldots, r_n)$ of $1, \ldots, n$. The test of $\mathbf{H}_0$ is then based on vector of linear rank statistics

$$\widetilde{\mathbf{S}}_n = n^{-1/2} \sum_{i=1}^{n} (\mathbf{x}_i - \bar{\mathbf{x}}_n) a_n(\tilde{R}_i). \tag{3.12}$$

The test criterion for $\mathbf{H}_0$ is the quadratic form in $\widetilde{\mathbf{S}}_n$,

$$\widetilde{\mathcal{T}}_n^2 = (A(\varphi))^{-2} \, (\widetilde{\mathbf{S}}_n)^\top \mathbf{Q}_n^{-1} \widetilde{\mathbf{S}}_n. \tag{3.13}$$

Because $\widetilde{\mathbf{S}}_n$ is distribution free under $\mathbf{H}_0$, the test based on $\widetilde{\mathcal{T}}_n^2$ has the same null distribution as the one based on $\mathcal{T}_n^2$, and their common distribution depends on the matrix $\mathbf{Q}_n$. Hence, their asymptotic null distributions are the same, and as such, they have the same critical region, which asymptotically can be approximated by the right hand tail of the χ^2 distribution with p degrees of freedom. Its asymptotic distribution under the local alternative (3.7) is noncentral χ^2 with p degrees of freedom and the noncentrality parameter

$$\Delta_{\tilde{H}} = \boldsymbol{\beta}^{*\top} \mathbf{Q} \boldsymbol{\beta}^* \, \frac{\gamma^2(\varphi, \tilde{H})}{A^2(\varphi)}$$

where $\tilde{H}$ is the distribution function of $\tilde{e}_i = e_i + \nu(\mathbf{Z}_i) + V_i$.

4 Rank analysis of covariance in partially linear models

Consider the model (1.1) as a partially linear model with possible measurement errors. If the Y_i are observed only with measurement errors, then these errors can be absorbed in the errors e_i of the model. More important is when the covariates $\mathbf{Z}_i$ are observed only with errors, hence we only observe $\mathbf{W}_i = \mathbf{Z}_i + \boldsymbol{\eta}_i$, $i = 1, \ldots, n$. Hence, model (1.1) can be rewritten in the form

$$Y_i = \beta_0 + \mathbf{x}_i^\top \boldsymbol{\beta} + e_i^{**} \tag{4.1}$$
$$e_i^{**} = e_i + \nu(\mathbf{W}_i), \quad \mathbf{W}_i = \mathbf{Z}_i + \boldsymbol{\eta}_i, \quad 1 \le i \le n,$$

where Y_i, $\mathbf{x}_i$ and $\mathbf{W}_i$ are all observable, but $\mathbf{W}_i$ and e_i^{**} may no longer be independent. Information on this dependence is recovered through the rank analysis of covariance approach, whose invariance structure enables to prevail this dependence, and even enhaces the power of the test of $\mathbf{H}_0$. A semiparametric approach estimating $\nu(\mathbf{W})$ nonparametrically, using a suitable smoothing tool, possibly leads to a slower rate of convergence; inference on $\boldsymbol{\beta}$ is then made in a parametric way.

Let $R_{ni}^{(j)}$ be the rank of W_{ij} among $W_{1j}, \ldots, W_{nj}$, $1 \le i \le n$; $1 \le j \le q$. Denote $\mathbf{W}_i = (W_{i1}, \ldots, W_{iq})^\top$, $1 \le i \le n$. Moreover, let $R_{ni}^{(0)}$ be the rank of Y_i among $Y_1, \ldots, Y_n$, $1 \le i \le n$. Denote

$$\mathbb{R}_n = \Big[\mathbf{R}_{n1}, \ldots, \mathbf{R}_{nn} \Big] \tag{4.2}$$

the $(q + 1) \times n$ rank collection matrix, where

$$\mathbf{R}_{ni} = \left(R_{ni}^{(0)}, R_{ni}^{(1)}, \ldots, R_{ni}^{(q)} \right)^{\top}, \ 1 \leq i \leq n.$$

Recall that under $\mathbf{H}_0 : \ \boldsymbol{\beta} = \mathbf{0}$ are $(Y_i, \mathbf{W}_i)^{\top}, \ i = 1, \ldots, n$, independent identically distributed $(q + 1)$-vectors, while Y_i and $\mathbf{W}_i$ are not necessarily independent. Denote $G^*(\mathbf{u}), \ \mathbf{u} \in \mathbb{R}^{q+1}$ the distribution function of $(Y_i, \mathbf{W}_i)^{\top}$.

Define a set of $(q + 1)$ scores $a_{nj}(k), \ 1 \leq k \leq n$ for $j = 0, 1, \ldots, q$, in the same manner as in Section 3. For the notational simplicity, we may take $a_{nj}(k) = a_n(k), \ 0 \leq j \leq q, \ k = 1, \ldots, n$. Define the random p-vectors

$$\mathbf{T}_{nj} = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} (\mathbf{x}_i - \bar{\mathbf{x}}_n) a_n \left(R_{ni}^{(j)} \right), \ \ 0 \leq j \leq q.$$

Define the matrix $\mathbf{V}_n$ of order $(q + 1) \times (q + 1)$ with the components

$$v_{nj\ell} = \frac{1}{n - 1} \sum_{i=1}^{n} \left(a_n \left(R_{ni}^{(j)} \right) - \bar{a}_n \right) \left(a_n \left(R_{ni}^{(\ell)} \right) - \bar{a}_n \right), \ \ j, \ell = 0, 1, \ldots, q.$$

Under $\mathbf{H}_0 : \ \boldsymbol{\beta} = \mathbf{0}$, the n columns of $\mathbb{R}_n$ in (4.2) are interchangeable with the common permutational (conditional, given the set of $n!$ possible realizations of $\mathbb{R}_n$) probability $\frac{1}{n!}$. Denoting this permutation measure $\mathcal{P}_n$, we have

$$\mathbb{E}_{\mathcal{P}_n} \mathbf{T}_{nj} = \mathbf{0}, \ \ \ \mathbb{E}_{\mathcal{P}_n} (\mathbf{T}_{nj} \mathbf{T}_{n\ell})^{\top} = v_{nj\ell} \mathbf{Q}_n \ \text{ for } \ j, \ell = 0, 1, \ldots, q$$

with $\mathbf{Q}_n$ being the matrix defined in (3.2). Decompose the matrix $\mathbf{V}_n$ in the form

$$\mathbf{V}_n = \begin{bmatrix} v_{n00} & \mathbf{v}_{n0}^{\top} \\ \mathbf{v}_{n0} & \mathbf{V}_{n11} \end{bmatrix} \tag{4.3}$$

and put

$$v_{n00.1} = v_{n00} - \mathbf{v}_{n0}^{\top} \mathbf{V}_{n11}^{-1} \mathbf{v}_{n0}, \tag{4.4}$$
$$\mathbf{T}_{n0:1} = \mathbf{T}_{n0} - (\mathbf{T}_n^*)^{\top} \mathbf{V}_{n11}^{-1} \mathbf{v}_{n0}$$

where

$$\mathbf{T}_n^* = (\mathbf{T}_{n1}^{\top} \ldots, \mathbf{T}_{nq}^{\top})^{\top}.$$

Thus, $\mathbf{T}_{n0:1}$ is the vector of residual rank statistics of Y_i's in the regression of $\mathbf{T}_{n0}$ on $\mathbf{T}_n^*$. Note that

$$\mathbb{E}_{\mathcal{P}_n} \mathbf{T}_{n0:1} = \mathbf{0},$$
$$\mathbb{E}_{\mathcal{P}_n} (\mathbf{T}_{n0:1} \mathbf{T}_{n0:1}^{\top}) = v_{n00.1} \mathbf{Q}_n.$$

This suggests the test criterion

$$\mathcal{L}_n^0 = \frac{1}{v_{n00.1}} \left(\mathbf{T}_{n0:1}^{\top} \mathbf{Q}_n^{-1} \mathbf{T}_{n0:1} \right)$$

which can be further rewritten as

$$\mathcal{L}_n^0 = \mathcal{L}_n - \mathcal{L}_n^*$$

where

$$\mathcal{L}_n = \mathbf{T}_n^\top \mathbf{Q}_n^{-1} \otimes \mathbf{V}_n^{-1} \mathbf{T}_n, \quad \mathcal{L}_n^* = \frac{1}{v_{n00}} \left(\mathbf{T}_{n0}^\top \mathbf{Q}_n^{-1} \mathbf{T}_{n0} \right).$$

Regarding the rank permutation distribution $\mathcal{P}_n$ described above, we conclude that the critical region of $\mathcal{L}_n^0$ can be obtained by enumerating the $n!$ possible permuted values of $\mathbb{R}_n$ and the corresponding values of $\mathcal{L}_n^0$. Due to the permutation invariance of the pertaining components, $\mathcal{L}_n^0$ is *permutationally distribution-free* [permutation principle of Chatterjee and Sen (1964)]. Asymptotically, as $n \to \infty$, the permutational distribution of $\mathcal{L}_n^0$ can be approximated by the χ^2 distribution with p degrees of freedom.

Under the local alternative (3.7),

$$\mathbf{V}_n \xrightarrow{p} \mathbf{\Gamma} = \left[\gamma_{j\ell} \right]_{j,\ell=0}^q \quad \text{as} \ \ n \to \infty, \tag{4.5}$$

the limiting rank score covariance matrix. Decompose $\mathbf{\Gamma}$ analogously as in (4.3),

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_{00} & \boldsymbol{\gamma}_0^\top \\ \boldsymbol{\gamma}_0 & \mathbf{\Gamma}_{11} \end{bmatrix} \tag{4.6}$$

and put

$$\gamma_{00.1} = \gamma_{00} - \boldsymbol{\gamma}_0^\top \mathbf{\Gamma}_{11}^{-1} \boldsymbol{\gamma}_0. \tag{4.7}$$

Note that the distribution of $\mathbf{T}_n^*$ does not depend on (3.7) [as the $\mathbf{Z}_i$ are i.i.d], and hence under (3.7) the shifts of $\mathbf{T}_{n0:1}$ and of $\mathbf{T}_{n0}$ coincide. Thus, under the local alternative (3.7)

$$\mathcal{L}_n^0 \xrightarrow{\mathcal{D}} \chi^2_{p,\Delta_H^*}, \tag{4.8}$$

the noncentral χ^2 with p degrees of freedom and with noncentrality parameter

$$\Delta_H^* = \boldsymbol{\beta}^{*\top} \mathbf{Q} \boldsymbol{\beta}^* \frac{\gamma^2(\varphi, H)}{\gamma_{00.1}}$$

with $\gamma_{00.1}$ defined in (4.7). This further implies

$$\gamma_{00.1} \leq \gamma_{00} = A^2(\varphi), \tag{4.9}$$

where the equality sign holds only when $\boldsymbol{\gamma}_0 = \mathbf{0}$; hence Δ_H^* cannot be smaller than the noncentrality parameter Δ_H (see (3.11)) of the analysis of variance rank test with the same score function. The asymptotic relative efficiency (ARE) of the analysis of covariance rank test relative to the analysis of variance rank test, based on the same score function $\varphi(.)$ is given by

$$ARE \text{ (ANOCOVA vs. ANOVA)} = \frac{\gamma_{00}}{\gamma_{00.1}} \geq 1 \ ; \tag{4.10}$$

hence, the analysis of covariance test is always at least as efficient as the analysis of variance test.

Summarizing, we conclude that the standard rank tests of linear hypothesis can be used even in the presence of a nonlinear nuisance regression or if there are measurement errors in the response or in the regressor, provided all these entities are i.i.d. and independent of each others and of the model errors. If we use the test while ignoring these disturbances, the probability of the error of the first kind is unchanged, while the disturbances only affect the power. If the nuisance regressors are observable, using the rank analysis of covariance still enhances the power.

4.1 Mixed linear model

Consider the mixed model

$$Y_i = \beta_0 + \mathbf{x}_i^\top \boldsymbol{\beta} + \mathbf{Z}_i^\top \boldsymbol{\gamma} + e_i, \tag{4.11}$$

where $\mathbf{Z}_i$, $i = 1, \ldots, n$ are stochastic q-vectors and $\boldsymbol{\gamma}$ is an unknown parameter. The mixed linear models with random and nonrandom covariates were studied in monographs by Khuri et al. (1998) and by Muller and Stewart (2006); the first one has a more theoretical flavor, while the second one focuses on detailed applications. However, if the random covariate is observed with an error, even the mixed linear model leads to the form (1.1) with a nonlinear nuisance regressor. Assume that the $\mathbf{Z}_i$ are not directly observable, but are subject to measurement errors $\boldsymbol{\eta}_i$, hence the observable random vectors are $\mathbf{W}_i = \mathbf{Z}_i + \boldsymbol{\eta}_i$. Assume that the $\boldsymbol{\eta}_i$ are independent of both $\mathbf{Z}_i$ and e_i. Without loss of generality assume that the $\mathbb{E}\mathbf{W}_i = \mathbf{0}$. Notice that Y_i and $\mathbf{W}_i$ are independent, given $\mathbf{Z}_i$. Hence, the conditional distribution function of $Y_i^o = Y_i - \beta_0 - \mathbf{x}_i^\top \boldsymbol{\beta}$ given $\mathbf{W}_i$, denoted by $f_{(y^o|\mathbf{w})}(y^o|\mathbf{w})$, can be written as

$$
\begin{aligned}
f_{(y^o|\mathbf{w})}(y^o|\mathbf{w}) &= \int_{\mathbb{R}^q} f_{(y^o,\mathbf{z}|\mathbf{w})}(y^o, \mathbf{z}|\mathbf{w}) \mathbf{dz} \\
&= \int_{\mathbb{R}^q} f_{(y^o|\mathbf{w},\mathbf{z})}(y|\mathbf{w}, \mathbf{z}) f_{(\mathbf{z}|\mathbf{w})}(\mathbf{z}|\mathbf{w}) d\mathbf{z} \\
&= \int_{R^q} f_{(y^o|\mathbf{z})}(y^o - \boldsymbol{\gamma}^\top \mathbf{z}) f_{(\mathbf{z}|\mathbf{w})}(\mathbf{z}|\mathbf{w}) d\mathbf{z}.
\end{aligned}
\tag{4.12}
$$

If the two conditional densities are Gaussian, then (4.11) corresponds to the linear measurement error model, with $\boldsymbol{\gamma}$ replaced by $\mathbf{K}^\top \boldsymbol{\gamma}$, where $\mathbf{K}$ is the matrix of $\boldsymbol{\Sigma}_\mathbf{z}(\boldsymbol{\Sigma}_\mathbf{w})^{-1}$. However, if the two densities are not Gaussian, then (4.12) involves a nuisance function $\nu(\mathbf{W}_i)$, where the form of $\nu(.)$ is unspecified, depending on the unknown densities. Note that here $\mathbf{W}_i$ are observable, not $\mathbf{Z}_i$, hence we will have $\nu(\mathbf{W}_i)$ instead of $\nu(\mathbf{Z}_i)$ in (1.1). Hence note that even for the mixed linear model (4.11) if the densities are not all Gaussian, we may not have a linear model, but based on (4.12), we can adapt a partially linear model as in (1.1). This enables us to incorporate rank analysis of covariance tests to have better power properties. There is, however, one salient point that we need to emphasize here. Since the rank ANOCOVA test is conditionally (permutationally) distribution-free, for small to moderate sample sizes the permutation distribution

needs to be enumerated to compute the permutational critical values. This task is quite manageable for small sample sizes but becomes prohibitively laborious as the sample sizes increase. Though for large samples, asymptotics work out well, for moderate to small sample sizes, to aid permutation distribution enumeration, classical resampling tools (such as jackknife or bootstrap methods) can be used. We refer to the next two sections for these refinements.

5   Numerical illustrations

In order to illustrate the proposed procedures for finite sample situation, we have conducted a simulation study.
We considered three semiparametric partially linear models

$$Y_i = \beta_0 + x_i\beta_1 + w_i\gamma + e_i, \quad i = 1, \ldots, n, \tag{5.1}$$

$$Y_i = \beta_0 + x_i\beta_1 + w_i^2\delta + e_i, \quad i = 1, \ldots, n, \tag{5.2}$$

$$Y_i = \beta_0 + x_i\beta_1 + \sin(w_i) + e_i, \quad i = 1, \ldots, n, \tag{5.3}$$

with $w_i = z_i + \eta_i$, $\quad 1 \le i \le n$, where η_i, $\quad 1 \le i \le n$ are measurement errors. The errors e_i, $i = 1, \ldots, n$, were simulated from the normal $N(0, 1)$, Laplace $L(0, 1)$ and Cauchy distributions, respectively. The measurement errors η_i, $\quad 1 \le i \le n$, were generated independently from the normal $N(0, 0.7)$, $N(0, 2)$ and uniform $U(-1, 1)$ distributions.

The design points $x_1, \ldots, x_n$ were generated from the uniform distribution on the interval (-2,10) and $z_1, \ldots, z_2$ from the uniform distribution on the interval (-10,30). They remain fixed for all simulations under given n.
The following parameter values of models were used:

- sample sizes: $n = 20, 100, 500$;

- $\beta_0 = 1$;

- $\beta_1 = -0.5, -0.4, \ldots, 0, \ldots, 0.4, 0.5$;

- $\gamma = 3$;

- $\delta = -2$.

Our interest is testing the hypothesis $\mathbf{H} : \ \beta_1 = 0$ against alternative $\mathbf{K} : \ \beta_1 \ne 0$. We use the test criterions $\mathcal{T}_n^2$ in (3.13) and $\mathcal{L}_n^0$ in (4.8). 10 000 replications of the models were simulated for each combination of the parameters and each distribution of measurement errors, and the test criterions were then computed for the Wilcoxon scores. The level $\alpha = 0.05$ test was performed every time, the mean power of the pertaining tests was then calculated. Figures 1–3 compare the powers in model (5.2) with standard normal distribution of errors e_i, $i = 1, \ldots, n$, for various sample sizes. We can see that results for small n, i.e. $n = 20$, are not overly good, but the results are much better for larger sample sizes. Comparing Figures 3 and 4 shows an effect of the distribution of errors e_i, $i = 1, \ldots, n$ in model (5.2) with $n = 500$. Figures 3,

5 and 6 compare the powers for different models, i.e. for (5.2), (5.1) and (5.3), for sample size $n = 500$. Figure 7 compares the empirical powers based on $\mathcal{L}_n^0$ and $\mathcal{T}_n^2$ in the models (5.2) and (5.3) for single size $n = 500$. Superimposing the power of the analysis of covariance rank test on the same for the analysis of variance rank test, we see that the analysis of covariance test performs better than the analysis of variance test in all cases; more prominently for large sample sizes and when the measurement error variance is not small compared to the error variance of the e_i. This is perfectly in line with our theoretical claim in (4.10). When the measurement error variance is small, the rank covariance ν_{01} is likely to be small too, and hence, this supremacy of the analysis of covariance test to the analysis of variance test is less perceptible for n = 20 (see Fig. 1 and 2). The picture becomes more pronounced for larger sample sizes (Fig. 4-7).

We have made more extensive simulation experiments. Particularly, various score functions, design vectors, other underlying distributions of the error terms and the measurement errors with small variance were considered. The results were very good for larger sample sizes, similar to Figures 3–6. Naturally, the results are considerably affected by the distributions of the error terms, but on the other hand, the influence of the measurement errors with small variances and of the function ν of the covariate z is not so substantial. Here, too, the analysis of covariance tests give better results.

6 Application to the precipitation dataset

The test described above is applied to a datasets of 1-day precipitation amounts. This application makes use outputs of coupled atmosphere and ocean general circulation models of the NOAA Geophysical Fluid Dynamics Laboratory. The outputs with the daily resolution are available in the form of transient climate change simulations carried out under increasing greenhouse gas concentrations according to prescribed emission scenarios over 1961-2100. Models have a horizontal resolution $2.5 \times 2.0°$ (longitude $\times$ latitude) for South America.

A variable of primary interest Y (precipitation) is modeled using additional covariates: the time index x and the southern oscillation index Z, which is calculated from the monthly or seasonal fluctuations in the air pressure difference between Tahiti and Darwin. The model under consideration has the form:

$$Y_i = \beta_0 + x_i \beta_1 + \nu(Z_i) + e_i, \quad i = 1, \dots, n,$$

where the Z_i are observable but probably with measurement errors, and $\nu(.)$ is unknown.

For each scenario gridpoint we tested the significance of time index, i.e. $\mathbf{H} : \beta_1 = 0$ against alternative $\mathbf{K} : \beta_1 \neq 0$. Table 6.1 summarizes results of testing for all 888 gridpoints and three scenarios.

Table 6.1. *Rejection and non-rejection of the null hypothesis at level $\alpha = 0.05$*
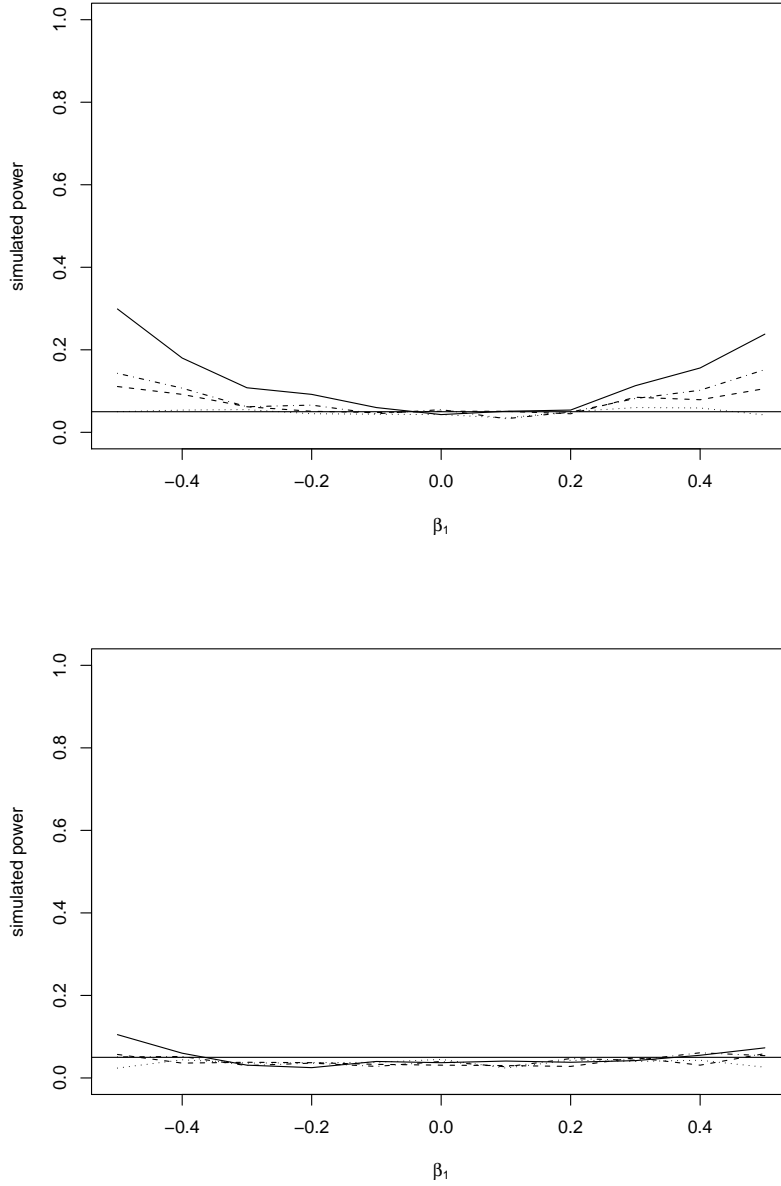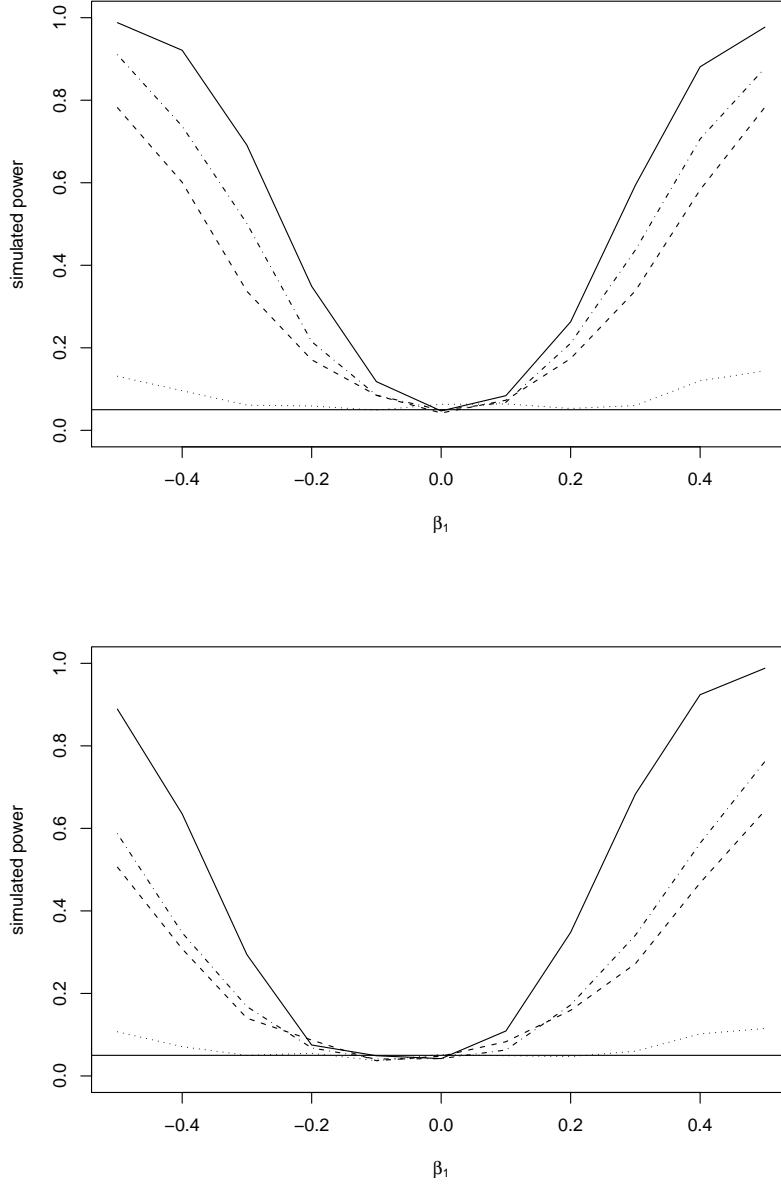
Figure 1: Empirical power of the Wilcoxon test based on $\mathcal{L}_n^0$(top) and $\mathcal{T}_n^2$ (bottom) for $n = 20$ in the model (5.2) under the standard normal errors e_i, $i = 1, \ldots, n$. Solid line corresponds to the standard test, i.e. $w_i = z_i$, $i = 1, \ldots, n$. The situations where $z_1, \ldots, z_n$ are affected by random errors are denoted by the dashed line (normal distribution $\mathcal{N}[0, 0.7]$), the dotted line (normal distribution $\mathcal{N}[0, 2]$) and dotdash line (uniform $U[-1, 1]$)
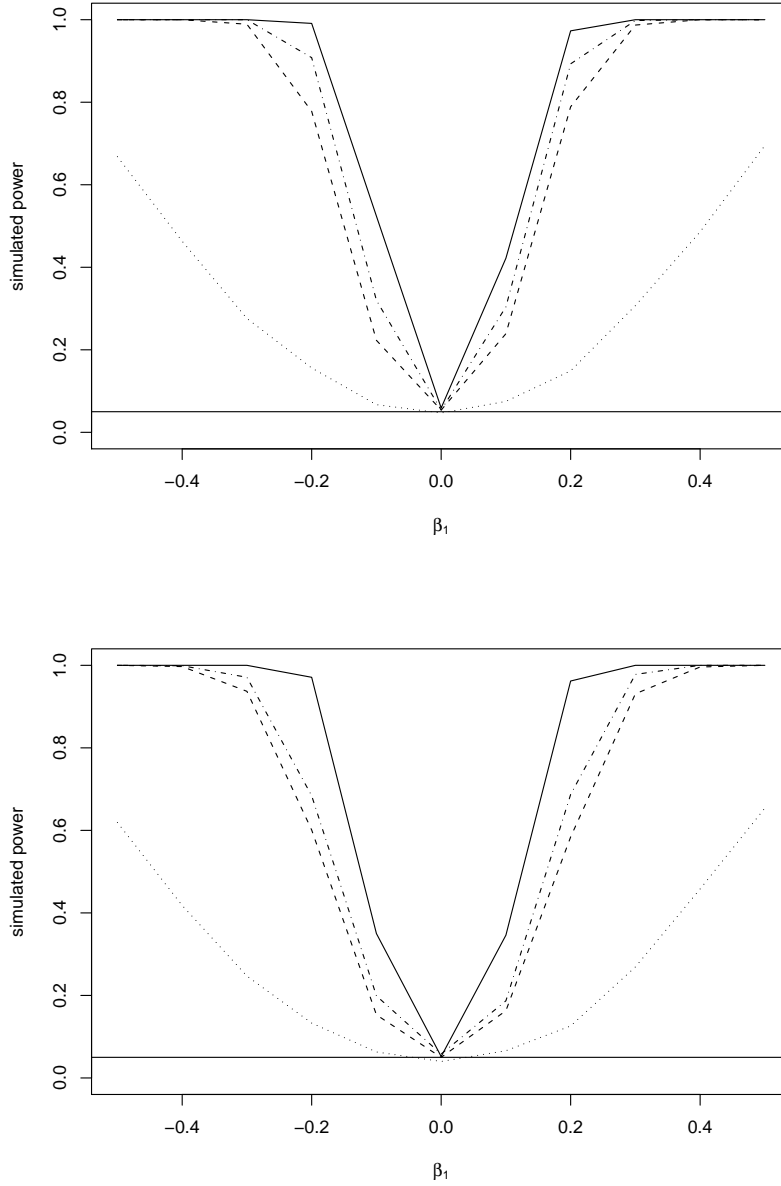
Figure 2: Empirical power of the Wilcoxon test based on $\mathcal{L}_n^0$(top) and $\mathcal{T}_n^2$ (bottom) for $n = 100$ in the model (5.2) under the standard normal errors e_i, $i = 1, \ldots, n$. Solid line corresponds to the standard test, i.e. $w_i = z_i$, $i = 1, \ldots, n$. The situations where $z_1, \ldots, z_n$ are affected by random errors are denoted by the dashed line (normal distribution $\mathcal{N}[0, 0.7]$), the dotted line (normal distribution $\mathcal{N}[0, 2]$) and dotdash line (uniform $U[-1, 1]$)
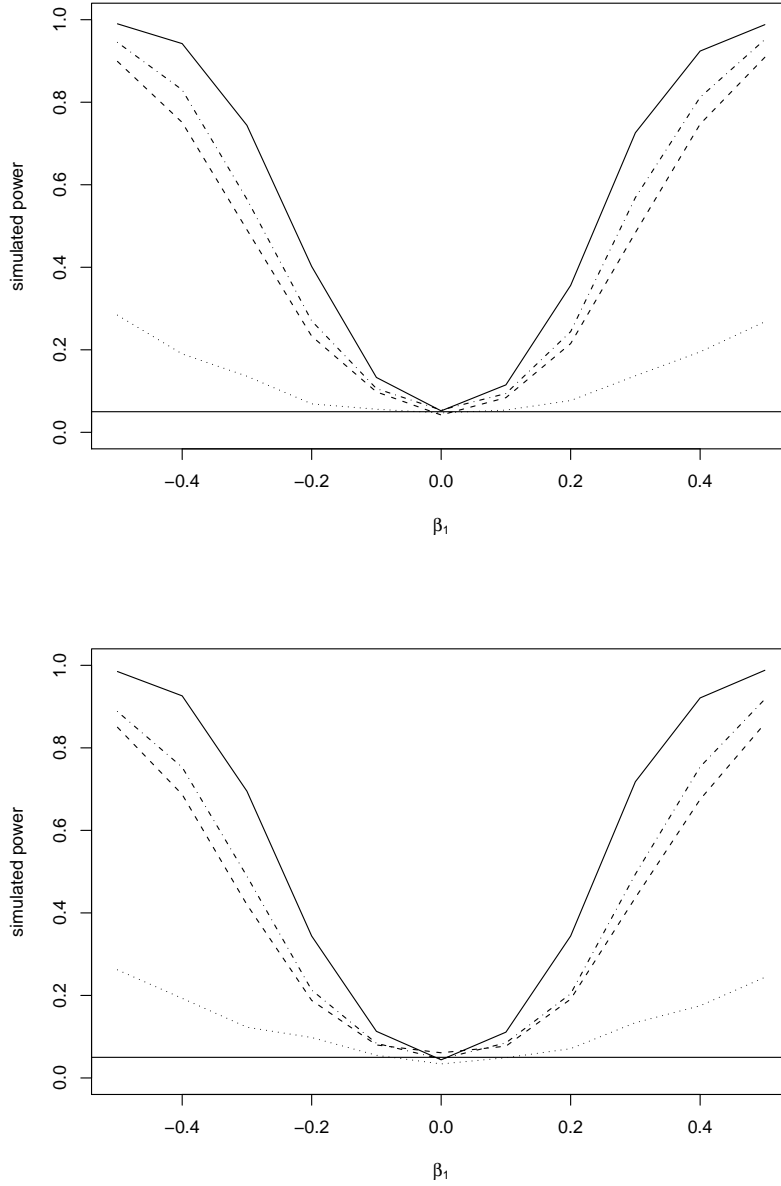
Figure 3: Empirical power of the Wilcoxon test based on $\mathcal{L}_n^0$(top) and $\mathcal{T}_n^2$ (bottom) for $n = 500$ in the model (5.2) under the standard normal errors e_i, $i = 1, \ldots, n$. Solid line corresponds to the standard test, i.e. $w_i = z_i$, $i = 1, \ldots, n$. The situations where $z_1, \ldots, z_n$ are affected by random errors are denoted by the dashed line (normal distribution $\mathcal{N}[0, 0.7]$), the dotted line (normal distribution $\mathcal{N}[0, 2]$) and dotdash line (uniform $U[-1, 1]$)

Figure 4: Empirical power of the Wilcoxon test based on $\mathcal{L}_n^0$(top) and $\mathcal{T}_n^2$ (bottom) for $n = 500$ in the model (5.2) under the Cauchy distributed errors e_i, $i = 1, \ldots, n$. Solid line corresponds to the standard test, i.e. $w_i = z_i$, $i = 1, \ldots, n$. The situations where $z_1, \ldots, z_n$ are affected by random errors are denoted by the dashed line (normal distribution $\mathcal{N}[0, 0.7]$), the dotted line (normal distribution $\mathcal{N}[0, 2]$) and dotdash line (uniform $U[-1, 1]$)
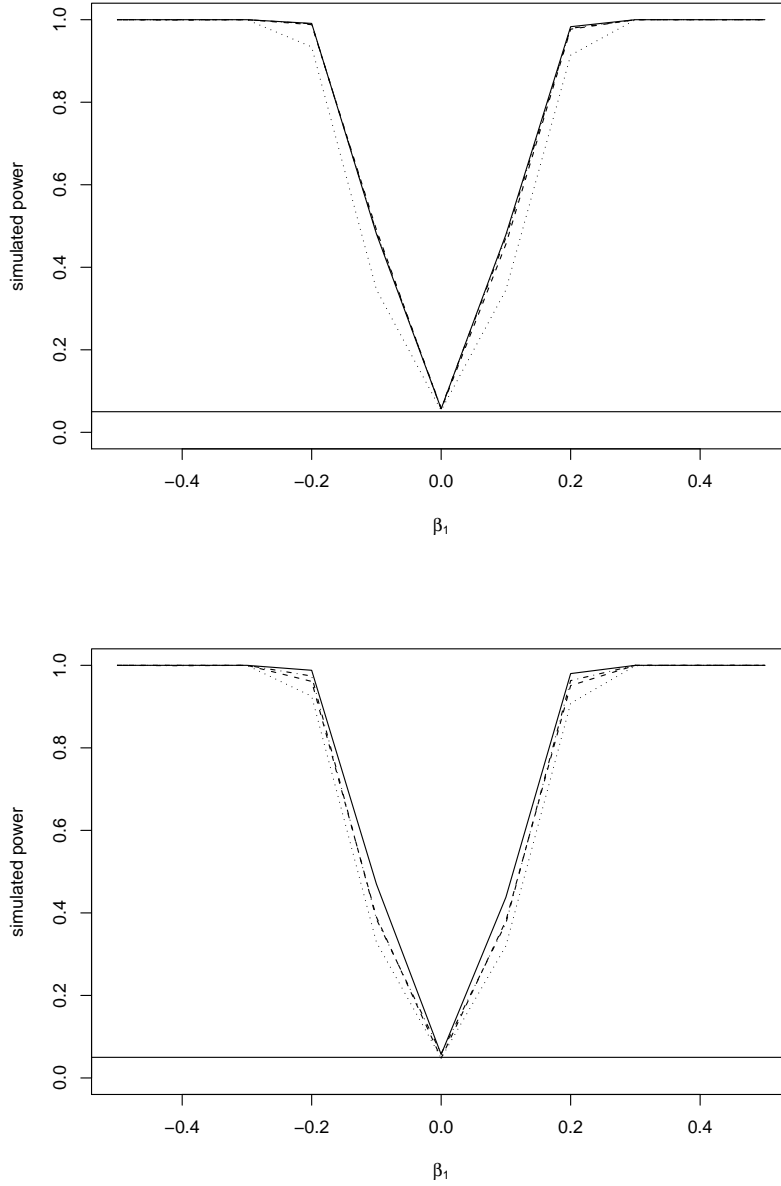
Figure 5: Empirical power of the Wilcoxon test based on $\mathcal{L}_n^0$(top) and $\mathcal{T}_n^2$ (bottom) for $n = 500$ in the model (5.1) under the standard normal errors e_i, $i = 1, \ldots, n$. Solid line corresponds to the standard test, i.e. $w_i = z_i$, $i = 1, \ldots, n$. The situations where $z_1, \ldots, z_n$ are affected by random errors are denoted by the dashed line (normal distribution $\mathcal{N}[0, 0.7]$), the dotted line (normal distribution $\mathcal{N}[0, 2]$) and dotdash line (uniform $U[-1, 1]$)
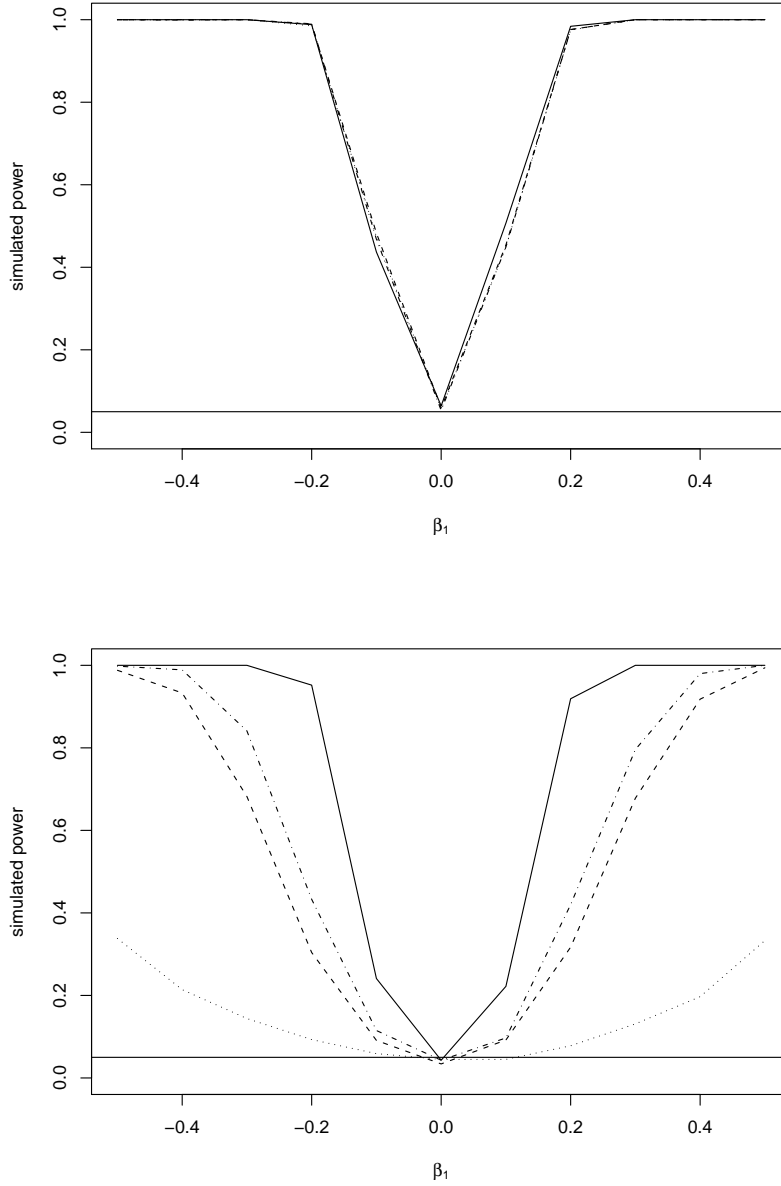
Figure 6: Empirical power of the Wilcoxon test based on $\mathcal{L}_n^0$ (top) and $\mathcal{T}_n^2$ (bottom) for $n = 500$ in the model (5.3) under the standard normal errors e_i, $i = 1, \ldots, n$. Solid line corresponds to the standard test, i.e. $w_i = z_i$, $i = 1, \ldots, n$. The situations where $z_1, \ldots, z_n$ are affected by random errors are denoted by the dashed line (normal distribution $\mathcal{N}[0, 0.7]$), the dotted line (normal distribution $\mathcal{N}[0, 2]$) and dotdash line (uniform $U[-1, 1]$)
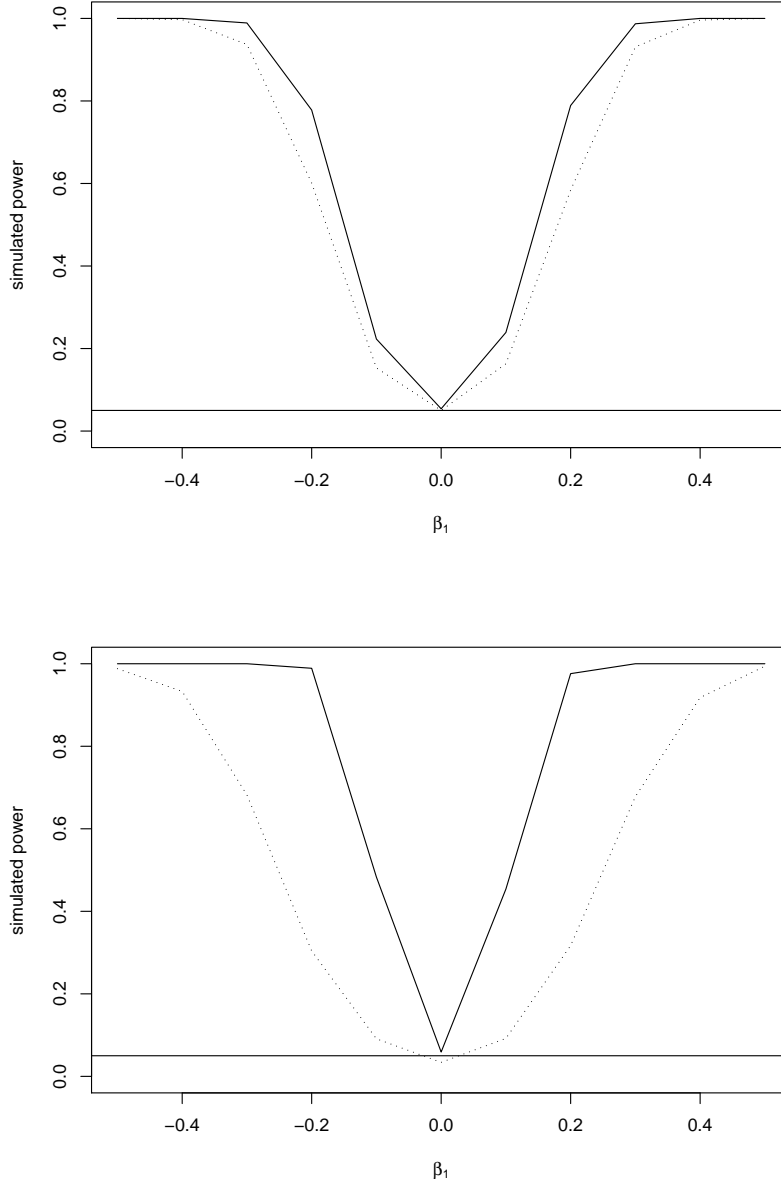
Figure 7: Comparison of empirical power based on $\mathcal{L}_n^0$ (solid line) and $\mathcal{T}_n^2$ (dotted line) for $n = 500$ in the models (5.2) (top) and (5.3) (bottom) under the standard normal errors e_i, $i = 1, \ldots, n$. The covariates $z_1, \ldots, z_n$ are affected by random errors coming from normal distribution $\mathcal{N}[0, 0.7]$

scenario	# of rejection $\mathbf{H}$	# of non-rejection of $\mathbf{H}$
scenario 1 (m21af)	465	423
scenario 2 (m21a2)	576	312
scenario 3 (m21b1	598	290

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